Description and commentary on the set of tables for the celestial navigation and for clearing the Lunar Distances published by David Thomson in 1824

In articles for the Internet discussion group "Nav-L" dealing with the problems of the celestial and non-electronic navigation (http://www.irbs.com/lists/navigation) that I have published to the theme of the clearing and using "Lunar Distances" (LD's), I mentioned the name of David Thomson many times. This person had fascinated me – the ordinary soldier on land and the deck hand at sea in the youth, he gained the position of a brig captain after many years of labor, but at the end, he died in Mauritius Island in the year 1834 as a lone storekeeper and didn't left any personal trace of himself.

However, in 1824 he published the set of navigation tables under the long title *Lunar and Horary Tables for new and concise Methods of performing the Calculations necessary for ascertaining the Longitude by Lunar Observations or Chronometers...* These tables were adapted for resolving the most difficult tasks of the celestial navigation of his time and made his name perennial. He had almost worked out the problem of clearing the Lunar Distances forever – his method (ranked among so called approximate methods for this task) was and is the shortest ever contrived. The method was unorthodox – Thomson constructed his main table empirically by comparing many and many cases of Lunar Distances computed by a rigorous method and the same LD's computed by the simplified method and tabulated the differences found in a large table (51 pages). A sailor could proceed by the same simplified and short method (even facilitated by other Thomson's tables, see below) and then only add the tabulated difference to his first result, instead of computing all values from scratch by long and bothersome trigonometric formulas. It is guessed that Thomson had to compute some 30000 lunar distances directly and to interpolate another 50000 values to construct his main table. Teachers of navigation and nautical astronomers of his time did not welcome such empirical way, of course, and Thomson's method never found place in official handbooks of navigation.

Nevertheless, it did find the way to decks of sailing ships. The first edition appeared in 1824, and each year approximately two editions followed. In 1851 the 42nd edition was published and in 1880 the 67th edition. Only Bowditch's *American Practical Navigator* experienced such long success in the history of the sea navigation, but this work had been enjoying great government support and generations of navigators kept and keep it up to date. To the contrary, after their first edition, Thomson's tables were published without any change by the same publishing house in London for decades and they died out only with the Lunar Distances themselves at the end of 19th century.

Although I am a passionate lover of David Thomson, I never held his tables in hands. However, George Huxtable, the member of the Nav-L group, succeeded in obtaining an old copy of them and sent its thorough and witty description to me. The tables themselves don't contain any factual explanations and formulae and give only succinct verbal rules for user, as was the habit of navigation manuals in the 19th century. It was a great delight for me to decipher their structure and to search for their basal formulae. As a result, I can publish this commentary on them, based on George's description and on some articles from old hydrographic journals of the 19th century. George's description follows as the first part of this article.

If you are interested in the role of Thomson's method in the history of Lunar Distances and its details, please read articles http://www.irbs.com/lists/navigation/0304/0048.html and http://www.irbs.com/lists/navigation/0304/0051.html. Take the present article as the third part of this mini series.

If you need and want to read more about Lunar Distances, George Huxtable gained the fame to be a guru for the matter of Lunar Distances (and others) and his treatise on them in the Nav-L group (in four parts) left very few questions open. Arthur Pearson, another member of Nav-L, has built the excellent website for instructing the modern (wo)men in the theme of LD's: http://www.lunardistance.com/. There you can find an introduction to the problem and many items concerning it (and links to four mentioned texts of George Huxtable, too).

1. The description of the set of Thomson navigation tables (by *George Huxtable*)

Remark, J.K. **– This description had been written as a friendly private help to me, not intended for publishing. Nevertheless, I had persuaded George to permit its publication here, as it was and is indispensable for the genesis, understanding and purpose of this whole article. I thank him very much. George's words follow:**

Here's a general idea of what the Thomson Tables contain (52nd ed., 1857).

27 tables altogether, some entirely trivial - (when I say 5 fig, this means 5 places for the mantissa, after the decimal point):

1. Longitude <---> time
2. Dip
3. Dip short
4. Augmentation of Moon semidiameter
5. Contraction of Moon semidiameter, due to refraction when being viewed at a skew angle in a lunar
6. Mean refraction
7. Corrections to refraction, allowing for nonstandard temperature and pressure
8. Interpolation between noon values for Moon semidiameter and HP
9. Best altitudes for measuring apparent time
10. Logs for interpolating sun dec. or RA

11. Log sec latitude and log cosec polar distance, 5 fig
12. Log cosines of the half sums of two angles, and the log sines of their differences, 5 fig. The index of each log has been reduced by 5, but as there are two such terms they will end up with a shortfall of 10, to be ignored.
13. Twice the log sines of half the respective horary angles (this is log hav under another name), 5 fig
14. Prop log of Moon's HP, lessened by 4600, 4 fig
15. Log cosec of apparent altitude (lessened by 9.5400, which corrects for the previous 4600, 4 fig
16. Log sin and log tan of apparent distance, rejecting 9 from the index. This makes it equivalent to log (10*sin dist), 4 fig
17. Logs of first and second corrections, 4 fig
18. Third correction (angle in minutes and seconds, to 1 second)
19. Proportional logs, 4 fig
20. Exactly the same as table 6
21. Correction of moon's apparent altitude (for parallax and refraction)
22. Logs of the Moon's apparent altitude (include corrections for altitude of sun or star), 6 fig
23. Logs of the sum of the apparent distance and the difference between the apparent altitudes of the objects. Also the log of the difference, which is what remains of the apparent distance when the difference in the apparent altitudes is subtracted therefrom, 6 fig
24. Logs of whole numbers 1000 to 9999, 6 fig
25. Natural versed sines, 6 fig
26. Time to be added to the RA of a star, to find the time of its passing the meridian on any day of the year (in hh.mm)
27. Correction to be subtracted from the observed alt of a fixed star to find the true alt to 0.1 min (This is star dip + refraction table)

Remark, J.K. **– Some tables are self-explaining and preparatory (Nos. 1-10, 20, 26, 27). Apart from them, there are three tables groups in the set, covering two basic procedures for determining the longitude in the first half of the 19^{th} century, i.e. for finding the local time and for clearing the Lunar Distances. These three groups will be commented on in the following text one after another.**

2. Thomson’s tables Nos. 11 – 13 for finding the local time and azimuths of celestial bodies

In a spherical / nautical triangle ABC / PZX take:

A ≡ P = elevated pole	B ≡ Z = zenith	C ≡ X = celestial body
α = t = local hour angle;	β = Az = Azimuth	γ = q = parallactic angle (not used)
a = z = zenith dist.	b = p = polar dist.	c = c = complement of latitude
h = 90° - z = altitude	δ = 90° - p = declination	φ = 90° - c = latitude

$$s = \tfrac{1}{2}(a+b+c)$$

$$s-b \quad = \tfrac{1}{2}(a+b+c)-b = \tfrac{1}{2}(a+b+c-2b) = \tfrac{1}{2}(z+p+c-2p) = \tfrac{1}{2}(90°-h-p+90°-\phi) =$$
$$= \tfrac{1}{2}(180°-h-p-\phi) = 90° - \tfrac{1}{2}(h+p+\phi)$$
$$s-c \quad = \tfrac{1}{2}(a+b+c)-c = \tfrac{1}{2}(a+b+c-2c) = \tfrac{1}{2}(z+p+c-2c) = \tfrac{1}{2}(90°-h+p-90°+\phi) =$$
$$= \tfrac{1}{2}(p+\phi-h) = \tfrac{1}{2}(p+\phi+h-2h) = \tfrac{1}{2}(p+\phi+h)-h$$
$$\sin^2 \tfrac{1}{2}\alpha = \operatorname{hav}\alpha$$

Thomson's tables Nos. 11 – 13 are constructed for the formulae:

$$\operatorname{hav} t = \csc p \sec\phi \cos\left[\tfrac{1}{2}(h+p+\phi)\right] \sin\left[\tfrac{1}{2}(h+p+\phi)-h\right]$$

and

$$\operatorname{hav}\left(180° - Az\right) = \csc p \sec\phi \cos\left[\tfrac{1}{2}(h+p+\phi)\right] \cos\left[\tfrac{1}{2}(p+h+\phi)-h\right]$$

George Huxtable has kindly transcribed Thomson's own verbal rules for me that were indispensable for understanding these procedures.

Table 11 is the simple log sec/csc table, table 12 is somewhat adapted log sine/cosine table, table 13 is the simple log haversine table. As two values are always taken from the table 12, their log characteristics are diminished by 5, to save subtracting 10 from the result. Such arrangements were Thomson's predilection.

Both formulae, as cited above, were deduced from the formulae for half angles in a common spherical triangle (see the symbols and auxiliary formulae above):

a) Local hour angle:

$$\sin\tfrac{1}{2}\alpha = \sqrt{\frac{\sin(s-b)\,\sin(s-c)}{\sin b \sin c}}$$

$$\sin^2\tfrac{1}{2}\alpha = \operatorname{hav}\alpha = \csc b\ \csc c\ \sin(s-b)\ \sin(s-c)$$

$$\operatorname{hav} t = \csc p\ \sec\phi\ \sin[90° - \tfrac{1}{2}(h+p+\phi)]\ \sin[\tfrac{1}{2}(h+p+\phi) - h]$$

$$\operatorname{hav} t = \csc p\ \sec\phi\ \cos\left[\tfrac{1}{2}(h+p+\phi)\right]\ \sin[\tfrac{1}{2}(h+p+\phi) - h]$$

(Thomson's formula for the hour angle)

And by the way:

$$\operatorname{hav} t = \csc p\ \sec\phi\ \cos\left[\tfrac{1}{2}(h+p+\phi)\right]\ \cos[90° + h - \tfrac{1}{2}(h+p+\phi)]$$

(Norie's formula for the hour angle)

This is an early case of the use of haversine (the first Thomson's edition took place in 1824!). Of course, the versine was used much earlier in double altitudes problem (Douwes), in rigorous methods for clearing LD's (Mackay, Krafft), and even for computing hour angles and reducing time sights (Bowditch's third method for obtaining the hour angle of a celestial body). However, the step to the haversine was done many years after in the practice of sea navigation, isn't it?

b) Azimuth:

$$\cos\tfrac{1}{2}\beta = \sqrt{\frac{\sin s \sin(s-b)}{\sin b \sin c}}$$

$$\cos^2\tfrac{1}{2}\beta = \csc b \csc c \sin s \sin(s-b)$$

and concurrently:

$$\cos^2\tfrac{1}{2}\beta = 1 - \sin^2\tfrac{1}{2}\beta = 1 - \operatorname{hav}\beta = \operatorname{hav}(180° - \beta)$$

therefore combined:

$$\operatorname{hav}(180° - \beta) = \csc b \csc c \sin s \sin(s-b)$$

and further, using symbols and auxiliary formulas from above:

$$\operatorname{hav}(180° - Az) = \csc p \sec\phi \sin\left\{90° - \left[\tfrac{1}{2}(p+h+\phi) - p\right]\right\}\cos\left[\tfrac{1}{2}(p+h+\phi)\right]$$

$$\operatorname{hav}(180° - Az) = \csc p \sec\phi \cos\left[\tfrac{1}{2}(p+h+\phi)\right]\cos\left[\tfrac{1}{2}(p+h+\phi) - p\right]$$

(Thomson's formula for the azimuth)

In this manner, Thomson had gained two formulae, one for the hour angle and another for the azimuth, which both required only three fully identical tables and coincided mutually but for one term (which changes in only one subterm "h" × "p" and in the function "sin" ×

"cos"). Both can be resolved simultaneously and side by side, if the sailor wanted to have the hour angle and azimuth for the same instant and the same celestial body. A rare deed, isn't it?

Of course, in 1824, when Thomson's tables were published for the first time, there was no use for azimuths of celestial bodies in LOP procedures – these didn't exist yet. However, aboard sailing ships the altitude-azimuth of a celestial body (gained from the altitude, latitude, and declination) was then badly needed for checking the compass. Amplitudes = azimuths of rising / setting Sun were not usable in the half of cases (statistically said) aboard sailing ships that ran constantly tilted under pressure of the wind. And usable methods for finding the time-azimuth (gained from the hour angle, latitude, and declination) were not at sailors' disposal up to the 2nd half of 19th century. All direct formulae that the nautical astronomy can offer for this task are too complicated for the everyday logarithmic use on deck. The inspection tables for turning up the time-azimuth (for the Sun and bodies with declinations of 23°N-23°S) were published by Burwood only in 1865 (for latitudes 30°-60°) and then by Davis in 1875 (for latitudes 0°-30°). Moreover, the universal time-azimuth ABC-Tables for any declinations and latitudes up to 75° appeared completed only after 1885. Hence, the Thomson's tables in their handy arrangement for finding the altitude-azimuth together with the hour angle were very useful in the first half of 19th century, when sailors were forced to measure magnetic bearings of the Sun during the forenoon altitude measurement and compare it with its computed altitude-azimuth (they had the accurate altitude for this moment!). Lecky calls this procedure obsolete only after 1880 – after Burwood's and Davis' azimuth tables had been published, but not knowing the ABC-Tables yet.

3. Thomson's tables Nos. 14 – 19 for clearing Lunar Distances by his own approximate method

These tables were most appreciated from the table set and made Thomson's name renowned. All are only 4-fig, as this was the sufficient accuracy for approximate algorithms of clearing LD's.

A remark: The name "the approximate method for clearing LD's" does not mean "an inaccurate method"! The word "approximate" concerns only the way of deriving the basic formula of the method, the results of "approximate" and "rigorous" methods for clearing LD's are (nearly, it should be said :) equally accurate. See the previous articles cited above.

Take the symbols as follows:

D = true (i.e. geocentric / to be compared with the tabulated Almanac values) lunar distance

d = apparent (i.e. observed) lunar distance

M = true (i.e. corrected for parallax and refraction) Moon's altitude

m = apparent (i.e. observed) Moon's altitude

S = true (i.e. corrected for parallax and refraction) Sun's / star's altitude

s = apparent (i.e. observed) Sun's / star's altitude

P = horizontal parallax of the Moon

The basis of the tables Nos. 14 – 19 is the formula:

$D - d = -P \sin s \csc d + P \sin m \cot d +$ Third Correction

This equation was gradually evolved by La Caille, Lexell, Maskelyne, and Lyons and introduced into the practice of sea navigation by Elford, Norie, and Thomson. (See my two previous texts about the classes of methods for clearing the LD's that I have sent to the Nav-L and mentioned with links at the beginning of this article.)

Actually, this formula is reciprocated for the construction of Thomson's tables, as the proportional logs are used for *P, d* and *D*. Prop-logs reciprocate the formulae, where they are present.

Table 14 gives proportional logs for possible values of *P* (53'0" – 61'59"), but diminished by 0.4600, therefore obtaining the table values 0.0710 – 0.0030 instead of 0.5310 – 0.4630. By this, Thomson wanted to simplify the possible table values and to diminish the number of necessary digits, as he did in a similar way on other places, too.

Table 15 is the cosecant table for altitudes within the range 5° – 88°59'. The values are enlarged by 0.4600 to allow for the same drop in the table 14. This addition changes nothing in using the values of this table and makes it not more difficult. Therefore, the facilitation from the table 14 is not lost. One should expect the sine table, not the cosecant one, according to the formula given above. This reciprocal is caused by the use of prop-logs, as mentioned above.

Table 16 gives the log sines and log tangents of *d* = apparent LD's within the range of 18° – 125°. In truth the values are "1 + sin *d*" and "1 + tan *d*", so that all values become greater than 1 and one should not fight with "9.xxxx" in log values. Again, the csc and cot from the basic formula were reciprocated in sin and tan by prop-logs.

Table 17 contains values of prop-logs (again enlarged by 1) for log sums of both first terms from the basic formula. (These sums were found by adding the values found in tables 14-16.) The values sought in the Table 17 are its upper and lower arguments. The upper arguments are equal to "$5° - P \sin h \csc d$"(the "First Correction", taken always from above), the lower arguments equal to "$5° + P \sin h \cot d$" (the "Second Correction", taken from below for $d < 90°$ and from above for $d > 90°$). The 5°, added twice, prevents the subtraction of a common value for the First Correction, see below. This table had been taken over by Bowditch as his Table XLVII.

Table 18 is the famous Thomson's table of the "Third Correction". I mentioned it at the beginning of this article and I wrote about its construction, accuracy, and peculiarities in my previous cited texts for Nav-L. I will not repeat my admiring words about it here. Its length (51 pages) prevented any tedious interpolation. It was the kernel of Thomson's tables set. It was taken over by Bowditch as his Table XLVIII and many times into other sets of navigation tables.

Table 19 is the commonplace table of proportional logarithms for 0-3 hours, facilitating the interpolation of GMT from the Almanac values of computed LD's.

At the end of the (relatively very short) computation, a sailor obtained his clearing correction to the observed LD by adding the 1st, 2nd (both from Table 17 after using Tables 14-16) and 3rd (from Table 18) corrections and subtracting 10° from the result (see the above remark about 5° that were added two times while using Table 17).

4. Thomson's tables Nos. 20 – 25 for clearing LD's by the adapted Dunthorne's rigorous method

From the Thomson's own verbal rules that I again owe to George Huxtable's thorough transcription, we can reconstruct these steps of this method:

Take symbols *D, M, S, d, m, s* as above and suppose $A = \dfrac{\cos M \cos S}{\cos m \cos s}$.

1) Make $m \sim s$; $M - m$; $s - S$

2) $(m \sim s) \pm \left[(M - m) + (s - S)\right] = M \sim S$

3) Make $d + (m \sim s)$; $d ;\sim (m \sim s)$

4) $\log Y = \log X + \log\left[d + (m \sim s)\right] + \log\left[d \sim (m \sim s)\right]$

5) $\operatorname{vers} D = Y + \operatorname{vers}(M \sim s)$

At the end of my searches, I have found that the basis of this procedure is the adapted Dunthorne's method:

$$\cos D = \cos(M \sim S) - A\left[\cos(m \sim s) - \cos d\right] \qquad \text{(established Dunthorne's formula)}$$

and as $\cos\alpha = 1 - \operatorname{vers}\alpha$, we can write:

$$1 - \operatorname{vers} D = 1 - \operatorname{vers}(M \sim S) - A\left[\cos(m \sim s) - \cos d\right]$$

$$\operatorname{vers} D = \operatorname{vers}(M \sim S) + 2A \sin\frac{d + (m \sim s)}{2} \sin\frac{d - (m \sim s)}{2}$$

By the way, this formula can be very simply adapted for haversines ($\frac{1}{2}\operatorname{vers}\alpha = \operatorname{hav}\alpha = \sin^2\frac{1}{2}\alpha$) and the result is the formula that in a different way deduced Bruce Stark for his newest and brilliant rigorous method for clearing LD's (1995/1997).

I will repeat the steps from above, inserting the remarks on tables:

1. Preparatory steps:

a) Create the difference *m~s*.

b) Find *S* from *s* and by the Table 20 (the mean refraction), which is repeated from the Table 6 to be at hand for this procedure. Create the difference *s-S*.

c) Find the apparent altitude of the Moon (*m*) in the interleaved tables 21+22. Find *M* from *m* and by the Table 21 (the correction for the combined effect of Moon's parallax and the mean refraction on Moon's altitude). Create the difference *M-m*.

2. Create the difference *M~S* by the formula $(m \sim s) \pm [(M - m) + (s - S)] = M \sim S$. (Change ± according to Thomson's rule: use +, if $m > s$, and use –, if $s > m$. The simple formula applies. I omit its proof.)

3. Create the differences $d + (m \sim s)$; $d \sim (m \sim s)$, omitting seconds of *d*.

4. This is a long step, at least in my words:

   a) The apparent Moon's altitude still lies open for you in the Tables 21+22. Table 22 should give logs of the value $X = 2A = 2\frac{\cos M \cos S}{\cos m \cos s}$, if I am not wrong, hence the doubled values of the well-known logarithmic differences, used nearly in all rigorous methods for clearing LD's (the method of Bruce Stark included) and tabulated many times after Maskelyne. Their characteristics are probably diminished by 6 by Thomson's typical trick, as the values 4.29xxxx (that stays at the beginning of nearly all values) hint. (Take the log of ordinary logarithmic difference $A = \frac{\cos M \cos S}{\cos m \cos s}$ for 30° and the Moon's horizontal parallax 53', add log of 2 and subtract 6: 9.996848 + 0.301030 – 6 = 4.297878. It should be your value of the Table 22 for Moon's apparent altitude 30° and its H.P. 53'.) The lost 6 from log characteristics should be allowed for in the subsequent Table 23 somehow.

      So, take the value log *X* from the Table 22.

   b) Take the log values for $d + (m \sim s)$; $d \sim (m \sim s)$ from the Table 23, being the log sine table with doubled angle arguments in view of the adapted Dunthorne's formula above. The characteristics of this table should be somehow tampered with to provide for the drop of 6 in characteristics of the Table 22.

   c) Sum the log *X* from the Table 22 and both log values taken from the Table 23, obtaining log *Y*.

   d) Convert log *Y* to natural value of *Y* by the Table 24, being the table of logs of whole numbers.

   e) Add *Y* and the value for (*M~S*) from the Table 25, being the table of natural versed sines.

5. For this sum, find *D* from the Table 25. Add the seconds omitted in the step 3 to it (I don't understand this detail very much, nor its core neither its purpose. Maybe one will be wiser from Thomson's examples.)

Of course, all these tables should be 6-fig, as this method for clearing LD's is rigorous.

Nevertheless, why it is published together with Thomson's clever and much shorter approximate method from the Tables 14-18? Maybe to give the user the choice, if he didn't have confidence in Thomson's Table 18 (the famous Third Correction), constructed empirically and not verifiable by the user.

Thomson himself says about his 2nd method for clearing LD's following: "*We shall now proceed to show the application of tables 20, 21, 22, 23, 24,and 25 to the clearing of apparent lunar distances from the effects of parallax and refraction. This operation cannot be performed by these tables in quite so short a time as it may by tables 14, 15, 16, 17, and 18; but it is perhaps the shortest and easiest method that has yet been proposed, on the same principle. It is also strictly correct, and applicable to all cases.*" (**According to George Huxtable.**)

I hope that I have contributed a bit to the fame of one of the most ingenious navigators in the sea history, who remains so enigmatic personality for us.

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