

The Accuracy of the Astronomical Observations of Lewis and Clark*

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HOW DID LEWIS AND CLARK KNOW WHERE THEY WERE?

THE FAMOUS expedition of Meriwether Lewis and William Clark up the Missouri River, over the Continental Divide, down to the Pacific via the Columbia River, and then back, lasted from May 1804 to September 1806. They kept crude track of their position using dead reckoning based on hand-held compass readings of their headings and visual estimates of distances covered. Their journals contain course and distance records such as:

S. 40° W ½ [mile] to some trees in a Stard. [starboard] bend

followed by the entry

S. 20° E 1 [mile] to some willows on the Lard. [larboard] Side

On 2 June 1805, the date of these entries, they made a total of nineteen course and distance notes.¹ Similar notes exist for every twist and turn of their route during all those months. Estimates of distance and compass headings were usually to the nearest quarter mile and the nearest five degrees, and were sometimes even more precise. Working with such notes and preliminary sketches, Clark made a series of route maps covering the whole trip. He, as well as others, later used these route maps and other information to create large-scale maps that included many important features of what is now the western United States (ref. 1, vol. 1).

However, it was well understood in advance that because of the unavoidable crudeness of the dead reckoning, sizable errors in the positions of points on the route maps would accrue as the journey pro-

*Five of the six principal figures connected with these measurements were members of the American Philosophical Society: Andrew Ellicott, Ferdinand Hassler, Thomas Jefferson, Meriwether Lewis, and Robert M. Patterson.

ceeded. Therefore, in accordance with their instructions from President Thomas Jefferson, who anticipated this problem, they carried navigational instruments with them, including a chronometer, a sextant, and an octant, all of the best quality available. If the true geographical positions—the latitudes and longitudes—could be determined for a succession of locations along the route, it would be possible to correct the route maps for much of the accumulated dead-reckoning error in both scale and orientation. This would improve the accuracy of the final large-scale maps.

Among Jefferson's instructions to Lewis were the following:

Beginning at the mouth of the Missouri, you will take observations of latitude and longitude, at all remarkable points on the river. . . .

and

Your observations are to be taken with great pains and accuracy; to be entered distinctly and intelligibly for others as well as yourself; to comprehend all the elements necessary, with the aid of the usual tables, to fix the latitude and longitude of the places at which they were taken; and are to be rendered to the war-office, for the purpose of having the calculations made concurrently by proper persons within the United States.²

The "usual tables" that Lewis and Clark took with them were three volumes of the *Nautical Almanac and Astronomical Ephemeris*, probably for the years 1803, 1804, and 1805,³ and Maskelyne's *Tables requisite to be used with the Nautical Ephemeris for finding the latitude and longitude at sea*.⁴

Lewis did complete one calculation of latitude and longitude from his own measurements at the mouth of the Missouri River, and attempted another at Fort Mandan. At Fort Mandan he also availed himself of a fortuitous lunar eclipse to try to establish the longitude of the place using measurements of a sort different from the usual ones. But aside from these efforts, surprisingly, the only recorded use he and Clark ever made of their many astronomical measurements was to determine latitudes. And until lately, apparently, no one else has ever used the Lewis and Clark data to calculate longitudes. Recently Robert Bergantino, who combines a long-standing interest in the Lewis and Clark expedition with considerable expertise in the theory and practice of celestial navigation, successfully calculated a longitude from Lewis and Clark observations.⁵

Traditionally, historians have supposed that this failure was due to the poor quality of the Lewis and Clark observations, or to the failure of Lewis and Clark to make all the required measurements at each

location, and have agreed with Donald Jackson's offhand dismissal of their astronomical observations as "highly suspect" due to "ineptness."⁶ Ineptness, however, is far from the explanation. The problem is that the data were taken in a somewhat unorthodox way to be used in calculations that some experts considered improper. Until Bergantino, apparently, no one who understood the unorthodox procedure ever tried to make the calculations.

HOW WERE ASTRONOMICAL OBSERVATIONS TO BE USED TO DETERMINE POSITION?

In the early nineteenth century, the principal tools of the navigator were a sextant (or an octant, which is a similar instrument), a clock or watch with a second hand, a copy of the *Nautical Almanac* for the current year, and a copy of the *Tables requisite*.

The sextant was used for two purposes. One was to measure the altitude of a celestial body, which is its height, in degrees, above the horizontal (Fig. 1). The other was to measure the angular separation (the lunar distance) between the moon and some other body such as the sun, a star, or a planet (Fig. 2).

The clock or watch was used to determine the time at which a sextant reading was taken. Unlike the chronometers used in more modern navigation, this timepiece was not expected to keep Greenwich time, but it was expected to gain or lose time at a known, fairly steady rate so that the time interval *between* measurements could be determined accurately.

The *Nautical Almanac* contained tables that allowed the navigator to determine the locations in the sky of the sun, moon, planets, and many stars for any instant of any day of the year for which that edition was valid.

The *Tables requisite* is a collection of tables of logarithms and trigonometric functions and some other specially devised tables designed to simplify the navigator's calculations.

At the time of the Lewis and Clark expedition, as at present, latitude was easily determined by measuring the altitude of the sun at its highest point in the sky—that is, at noon. By combining this value of the altitude with information tabulated in the *Nautical Almanac* for that year and day, the latitude was found by a simple calculation. Lewis and Clark made such measurements frequently and usually calculated the latitude soon afterward.

If the navigator knows the Greenwich time at the instant of noon, he can also determine his longitude from a simple calculation. The immediate difficulty is that it is hard to identify the instant when the

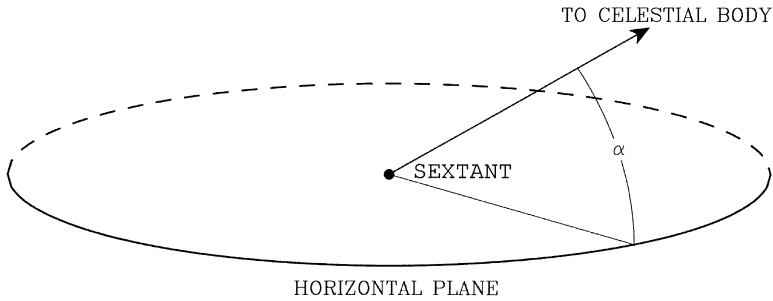


FIGURE 1. The horizontal plane containing the sextant is depicted here. The altitude α of a celestial body is defined as the angle between this plane and the direction of the celestial body. An observer at sea actually measures the angle between the horizon and the direction of the celestial body. To obtain α this measured angle has to be corrected for the fact that the horizon is slightly below the horizontal, depending on how high the sextant is above the water. Lewis and Clark, however, were on land, so they used an artificial horizon, which obviated this problem.

sun is at its maximum altitude because the vertical component of the sun's apparent motion is smallest near noon, and the sun stays near its highest altitude for quite a while. This is not a problem for determining latitude, but it produces a considerable uncertainty in the calculated longitude.

A surveyor or explorer on land can use an indirect method, known as the equal-altitudes method, to determine what his clock or watch *would* have read at noon if he had been able to identify that instant. Lewis and Clark often made equal-altitudes measurements, and they

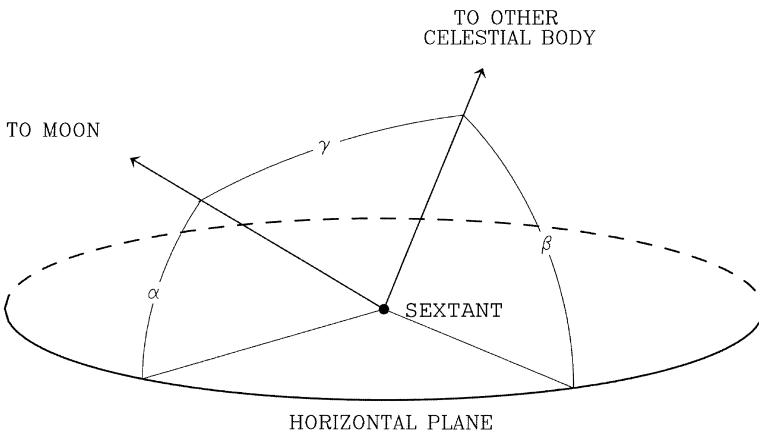


FIGURE 2. The angles α and β are the altitudes of a star and the moon. The lunar distance measured by the sextant is the angle γ between the moon and the star.

did so in the following way. About four hours before noon, one of them measured the sun's altitude and recorded it along with the watch time. About eight hours later, as the sun was going down toward the horizon, the observer noted the watch reading at the instant the sun came back down to exactly the same altitude as at the morning reading. The time halfway between the watch times of the morning and afternoon equal-altitude observations was then taken to be the watch time of local noon (after a small correction required because the sun itself would have drifted slightly to the north or the south—depending on the time of year—between the two readings).

A further difficulty in determining the Greenwich time of noon was that chronometers that would keep Greenwich time reliably over months and years were scarce and expensive. This would not have been a serious problem if the navigator had had a fairly good watch and could check it against Greenwich time every so often, as one can do now with a radio time signal. But even though a time check actually was available, as will be described, it was important to know how fast or slow the watch was running (its "rate") so that the Greenwich time at local noon could be calculated correctly from the difference in watch readings between noon and the time check. The rate at which the watch was gaining or losing time was found by making equal-altitudes measurements on successive days and noting the elapsed watch time between two successive noons. (The time from noon to noon varies throughout the year, but the *Nautical Almanac* contains the information necessary to allow for this.) Typically the rate was some number of seconds per day, either gained or lost. Once the rate was known, the Greenwich time of local noon could be determined from the Greenwich time of the time check and the difference between the watch reading at the instant of the time check and the watch reading at noon.

The only remaining question is how to obtain a time check.

THE TIME CHECK

Before there were chronometers that could keep Greenwich time accurately enough over the months or years that a mariner or explorer might be away from civilization, the time check required for finding longitude had to be obtained from other astronomical measurements. One of the best astronomical clocks available on a nearly daily basis for at least part of each year consists of the principal satellites of Jupiter. The disappearances and reappearances of Jupiter's satellites, as their orbital motions carry them around that planet and into and out of view, occur rather often and are predictable. A good astronomical telescope and the appropriate astronomical tables provided frequent

opportunities to determine Greenwich time fairly accurately by the satellites of Jupiter. Unfortunately for mariners, the chaotic motions of ships on the high seas prevented the use of astronomical telescopes, which have to be held very steady for these observations. Inland explorers on foot or in small boats could use dry land as a steady platform, but few of them carried good observing telescopes, which are bulky and fragile.

Other occasional but predictable events, such as eclipses of the moon or sun, can also be used to determine Greenwich time.

THE METHOD OF LUNAR DISTANCES (THE “METHOD OF LUNARS”)

A more convenient astronomical clock, although difficult to read with sufficient accuracy, is the moon. As viewed from the earth, the moon does not move through the sky at the same rate as the fixed stars. It falls behind by about 13° per day, and, moreover, the path it follows across the sky drifts northward or southward by noticeable amounts from day to day. The seemingly irregular motion of the moon against the background of the fixed stars has, in fact, become very predictable, thanks to Isaac Newton and his successors. This means that at any given instant the angular separation of the moon from any star or planet or from the sun has a definite, predictable value. By measuring this separation (the lunar distance) and using appropriate astronomical tables, the Greenwich time of the lunar-distance measurement may be determined. Because of this, for many years each of the annual editions of the *Nautical Almanac* had tables showing lunar distances from the moon to a selection of stars (including the sun) for every third hour of each day of the year. With this information the navigator could determine by interpolation the Greenwich time that corresponded to his measured lunar distance. This was his time check.

As outlined above, a typical day of observations for latitude and longitude would consist of the following steps:

- About four hours before noon, measure and record the altitude of the sun and record the watch time.
- At noon, measure and record the altitude of the sun.
- Calculate the latitude from the noon altitude.
- About four hours after noon, observe the altitude of the sun and record the watch time when the sun reaches exactly the same altitude as recorded at the morning measurement.
- Calculate the watch time of noon as the time halfway between the watch times of the morning and afternoon equal-altitudes measurements, corrected as necessary for the rate of the watch, and for the

slight drift of the sun north or south (depending on the season) between the two measurements.

- Whenever it becomes possible (before, between, or after the equal-altitudes measurements), measure the lunar distance to a chosen celestial body. Record the lunar distance along with the watch time of the measurement.
- Determine the Greenwich time of the lunar-distance measurement by comparing the lunar distance with data derived from the *Nautical Almanac*.
- Use the Greenwich time of the lunar-distance measurement with the difference between the watch time of the lunar-distance measurement and the watch time of noon (corrected for the rate of the watch) to determine the Greenwich time of noon.
- The Greenwich time of noon leads directly to the longitude, as stated earlier.

We now come to an important point that will explain why the Lewis and Clark observations were considered useless by some. The observed lunar distance cannot be used *directly* with the *Nautical Almanac* to determine the Greenwich time of the reading. The reason for this is refraction and parallax.

Parallax is a perspective effect. The moon is much closer to the earth than the fixed stars so at any instant its apparent position among the fixed stars depends slightly but significantly on where the observer is on earth. All the data in the tables of the *Nautical Almanac* are presented as if measured by an observer at the center of a transparent earth. For a fictitious observer at that location, by convention, the parallax is taken to be zero. Real observers, however, are necessarily at the *surface* of the earth, so all measurements involving the position of the moon have to be adjusted for parallax before being compared with numbers in the *Nautical Almanac*. Position measurements of the sun and the planets, and even the fixed stars, are also subject to parallax, but those bodies are so far away that neglect of this fact is a relatively minor source of error and can be ignored in determinations of longitude by the lunar-distance method.

The refraction effect is the downward bending of light rays coming to the earth from outer space as they pass through the atmosphere. Refraction makes any celestial object appear to be higher in the sky than it would if the earth had no atmosphere. The tables in the *Nautical Almanac* are calculated as if there were no refraction. Therefore the sextant altitude⁷ of any celestial body has to be corrected for refraction and possibly for parallax before the tables can be used. Only the sextant altitude needs such corrections, and these corrections depend only

on the sextant altitude itself. The sextant altitude of the moon will always be significantly too low, and the sextant altitude of anything else will always be slightly too high.

These vertical shifts in observed altitudes, being of different amounts and in different directions, change the apparent angular separation of the moon and the other body. Obviously, then, the sextant lunar distance⁸ has to be corrected for both refraction and parallax (Fig. 3).

In the early nineteenth century there were widely known procedures for calculating the corrected lunar distance,^{4,9-12} but for any such calculation the sextant altitudes of the moon and the other body at the instant of the lunar-distance measurement are needed, as well as the sextant lunar distance itself. This is an extra complication because, in principle, the lunar distance and the two altitudes must be measured simultaneously, or as nearly so as possible. Thus, either two more observers and two more sextants must be present, or the single observer must measure both altitudes in succession just before and again just after the lunar-distance measurement and then take averages of the before and after readings. Then, finally, the observed lunar distance can be corrected and the lunar-distance tables in the *Nautical Almanac* used to determine Greenwich time.

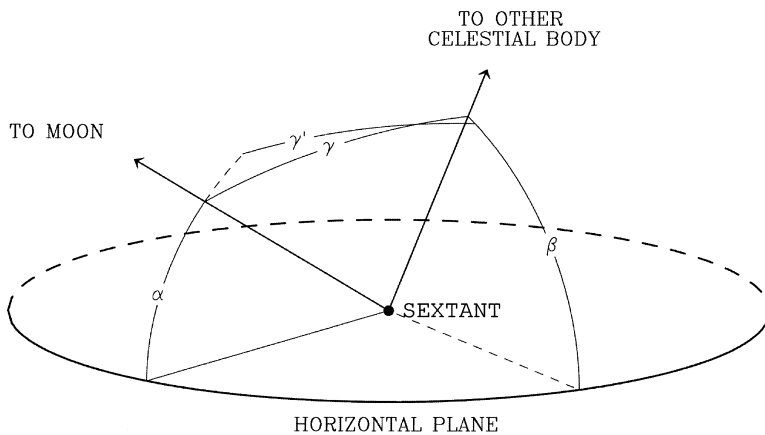


FIGURE 3. Using the values of α , β (from Fig. 1), along with small corrections to α and β for refraction and parallax, the corrected lunar distance γ' is calculated by spherical trigonometry. The corrected lunar distance is then compared with values in the *Nautical Almanac* to determine Greenwich time at the instant of the measurement. Because Lewis and Clark never measured α and β when they measured γ , Ferdinand Hassler was unable to calculate any γ' values and could not go on to determine the Greenwich times needed for computing any of their longitudes (see text).

Thus, for observers on land in the early nineteenth century, the most common and most trusted version of the method of lunars for determining longitude required

- Morning and afternoon equal altitudes of the sun
- Noon altitude of the sun
- A lunar distance and two altitudes, all measured simultaneously, if possible

WHY WEREN'T THE LEWIS AND CLARK LONGITUDES CALCULATED?

At first glance it is difficult to understand the neglect of the Lewis and Clark astronomical observations. They always made their own determinations of latitude from their noon altitude measurements, calculating and recording their results on the same day. This was a relatively simple calculation. It is the measurements for longitude that have, until recently, languished completely unused. Certainly Lewis and Clark had more pressing things to do than to spend time on long, finicky calculations of longitude while on their journey. In fact, in view of Jefferson's instruction to turn over their data to the war office for calculation by "proper persons," it seems clear that Jefferson did not expect them to bother with longitude calculations at all while en route. In this he was following the advice of one of his navigation experts.¹³

Inspection of the astronomical data recorded in the Lewis and Clark journals¹⁴ reveals that in *no* case did they make (or at least record) the required altitude measurements. In his journal, Lewis described the various artificial horizons he used for measuring the altitudes of the sun, moon, and stars,¹⁵ but although the two men made many sets of measurements of lunar distances from such bodies as the sun, Spica, etc., *not once did they record the altitude of the moon or of the other body at the time of the lunar-distance measurement.* It was the total absence of these altitude measurements in the Lewis and Clark data that prevented the calculation of their longitudes by the person who was appointed to carry out this task.

Why did Lewis and Clark fail to take and record the altitudes some considered essential for the method of lunars? Before setting out for the west, Lewis took a cram course on celestial navigation with the mathematician and astronomer Robert Patterson and with the astronomer and surveyor Andrew Ellicott, both of whom were prominent members of the American Philosophical Society. Patterson had created an astronomical notebook, which gives detailed instructions for the standard method of lunars much as I have described it. However, according to his instructions,¹⁶ "As it may frequently happen that the

alts. [altitudes] of one or both of the bodies cannot well be taken you must then *compute* the app. [apparent] alts by prob. 4th, *and this method is generally to be preferd* [sic] *on land* [emphasis added].”

Patterson’s Problem 4th, which is also a part of his notebook, describes a method of *calculating* the altitude of a body at any instant using the known latitude of the observer; the *estimated* longitude; a Greenwich time for that instant calculated from the estimated longitude and the difference between the watch time of the instant and the watch time of noon (with a correction for watch rate).

Although this is a reasonable procedure and gives excellent results, it seems to have been controversial at the time. Indeed, at first sight there appears to be an element of circularity in a procedure that starts with an *estimated* longitude and a corresponding estimate of the Greenwich time, and derives from them first the *actual* Greenwich time and then the *actual* longitude using no observational data except the latitude, an observed lunar distance, and two watch readings whose relation to Greenwich time is unknown.

Some works on navigation from that period show how to calculate the required altitudes, and at least one does not. Maskelyne’s *Tables requisite*,⁴ Problem VII, describes the calculation of an altitude using an estimated longitude without explicitly recommending the use of calculated altitudes for correcting lunar distances. Somewhat further on in the text, in Problem XI, the correction of lunar distances is described but, again, without mention of the possibility of using calculated altitudes for this purpose. John Robertson’s *Elements of Navigation*¹¹ gives a procedure for correcting lunar distances using calculated altitudes, as does John Moore’s *New Practical Navigator*¹² as well as Nathaniel Bowditch’s *New American Practical Navigator*,⁹ which used Moore’s work as a model.

Bowditch advises against calculating the altitudes, suggesting that the results are not accurate enough, but none of the other authors discusses the validity of the results. As late as the fifth edition (1822), P. Kelly’s *Practical Introduction to Spherics and Nautical Astronomy*¹⁰ does not even mention calculation of the required altitudes. On the other hand, Edward Riddle’s 1824 *Treatise on Navigation and Nautical Astronomy*¹⁷ does recommend calculation of the required altitudes, but only for night observations. For his comprehensive *History of Nautical Astronomy* (1968),¹⁸ Charles Cotter must have read widely in the literature on lunar distances. That he barely mentions the possibility of calculating the altitudes suggests that he believed the method was used rarely, if at all, and that he was himself skeptical of the validity of the method, as many navigators would be on first acquaintance with it.

Clearly, Robert Patterson understood and believed in the method.

Likewise, Andrew Ellicott, who was Lewis's other navigation instructor, had demonstrated his use of it in a self-congratulatory entry for 22 September 1799, in the appendix of his *Journal of Andrew Ellicott*.¹⁹ Likewise, Philip Turnor²⁰ and David Thompson,²¹ who were exploring western Canada in the years before, during, and after the Lewis and Clark expedition, used this very method of supplying the missing altitudes, and their results have turned out to be very reliable.

Lewis seems to have learned and understood the method very well. In a notebook entry (Codex O) for 22 July 1804, he mentioned a series of lunar-distance measurements he made while at the mouth of the Missouri River in December 1803. He calculated both the latitude and longitude, but, uncharacteristically, he entered the results of the calculation but not the measurements. As he did not have the three sextants or octants needed for the required three simultaneous measurements, it is likely that he used the Problem 4th method recommended by Patterson, or a similar method recommended by Ellicott. In a later field note²² Lewis initiated a lunar-distance calculation based on his sun-moon observations of 23 February 1805, while at Fort Mandan. His calculation breaks off at the point where he has calculated the altitude of the sun's center "by Mr. Elicot's formula" and "by Mr. Patterson's," but before he has calculated the moon's altitude. (It is interesting that the two calculated altitudes differ by more than a degree, but it is probable that if both required altitudes were calculated using the same formula, the final results for the longitude would have been practically the same for both formulas.)

In any case, it appears that all the rest of the Lewis and Clark measurements were made with the understanding that calculated altitudes would be used to calculate longitudes from their data. In the next section I describe Patterson's Problem 4th method. Unfortunately, while calculation of the required altitudes simplified the measurements, it made the calculations themselves harder. Lewis and Clark were wise to adopt this method of measurement and wise not to try to do the calculations themselves while en route.

WHAT WAS THE PROBLEM 4TH METHOD?

As previously mentioned, the Problem 4th calculation required (1) previous knowledge of the latitude as determined in the usual way by a noon altitude measurement, (2) the watch time of noon as determined from the equal-altitudes measurements, and (3) the watch time of the lunar-distance measurement. The procedure was then as follows:

- Adopt a reasonable estimate for the longitude.

- Using this estimated longitude, calculate the corresponding *estimate* of the Greenwich time of the lunar-distance measurement using this longitude and the difference between the watch time of noon and the watch time of the lunar-distance measurement (corrected for watch rate).
- Use the estimated longitude, the corresponding estimate of the Greenwich time of the lunar-distance measurement, the measured latitude, and the *Nautical Almanac* tables to calculate the altitudes of the moon and the other body at the instant of the lunar-distance measurement. (These altitudes are only approximate, of course, because they are based on an *estimate* of the longitude, but they happen to be extremely good approximations.)²³ Also, these altitudes are estimates of the altitudes *after* correction for refraction and parallax.
- “De-correct” these altitudes for refraction and parallax to obtain the sextant altitudes—the altitudes *before* corrections for refraction and parallax.
- Use these approximate altitudes in the usual way, *as if they were exact*, to calculate the corrected lunar distance. (Although based on an estimated longitude, this corrected value will be surprisingly close to the properly corrected value if the estimate used for the longitude is fairly reasonable.)
- Use this corrected lunar distance and the *Nautical Almanac* to obtain a final and much better estimate of Greenwich time at the instant of the lunar-distance measurement.
- Use this improved value of the Greenwich time of the lunar-distance measurement with the difference between the watch time of noon and the watch time of the lunar-distance measurement (corrected for the rate of the watch) to work back to the Greenwich time at noon.
- Use the Greenwich time at noon to calculate the actual longitude by the usual method. (This is the final value of the longitude.)

This looks very much like a single iteration of a procedure recently used by Bergantino⁵ to calculate a Lewis and Clark longitude. Bergantino, however, suggests using these results as initial guesses for a second round of calculations to obtain even better estimates of the Greenwich time and the longitude. Such successive approximations can be repeated until there is no discernible difference between the input and output values of the longitude and time.

I have written a computer program that carries out Bergantino’s iterative procedure, and I find that the results converge rapidly. Even when the initial estimate of the longitude is off by several degrees, the

final value is reached after three iterations, while the longitude calculated in the first iteration is usually within a few minutes of the final value. Therefore, in practice, the longitude obtained from the first iteration should be acceptable as “good enough,” considering that thirty-minute errors in the calculated longitude are not unusual for the lunar-distance method.

Like Bergantino, I find that the longitudes I calculate from Lewis and Clark data agree quite well with the modern values for the longitudes of their respective locations. Thus, Bowditch’s reservations about this method do not seem well founded, and the use of the method by Robert Patterson, Andrew Ellicott, Philip Turnor, David Thompson, Lewis and Clark, and probably many others, was quite appropriate.

Not content with this departure from the standard method of lunars, Andrew Ellicott developed an advanced version of it. In a letter to Jefferson dated 6 March 1803,¹³ he stated that he could dispense with the noon altitude measurement to determine the latitude and that he could get both the latitude and the longitude from just the morning and afternoon equal-altitudes measurements and the lunar-distance measurement.

A navigator who understood the modern “lines of position” method of determining both latitude and longitude from two altitude measurements would see immediately how this could be done. But since the position-line method was not discovered until 1837,²⁴ it is unlikely that Ellicott had any such thing in mind.

After I read Charles Cotter’s *History of Nautical Astronomy*, it occurred to me (ref. 18, p. 271) that Ellicott was resorting to the method of double altitudes, which is a means of determining the latitude directly using altitude measurements on the sun made at two different times (not necessarily equal-altitudes measurements) and the elapsed time between them. No noon altitude measurement is necessary. When two such readings have been made (the two equal-altitudes measurements can be used for this second purpose as well as for finding the watch reading at noon), the results can be used with information from the *Nautical Almanac* to determine the latitude. This, however, requires tedious trigonometric calculations, or a repetitious successive-approximations procedure, or a complicated graphical procedure involving stereographic projections. The difficult double-altitudes calculation for the latitude was used only infrequently and reluctantly at that period. This is why, following standard practice, Lewis and Clark usually made high-noon measurements of the sun’s altitude, as well as equal-altitudes measurements. The latitude could then be determined without having to make the formidable double-altitudes calculation. But if, for any reason, the noon altitude of the sun could not be measured,

the latitude could still be found using the double-altitude method as a last resort.

I have used computer programs to calculate latitudes as well as longitudes from twenty-three different sets of Lewis and Clark observations using the Problem 4th method both by itself and with Ellicott's 1803 extension of it. As can be seen from the accompanying table, both methods work well. All but three of the longitudes obtained from moon-to-sun lunar distances agree with the modern values to within a half a degree, which is about what is expected from good lunar-distance determinations.

Before the advent of computers, any of my calculations would have had to be done by hand. These calculations are ideal for solution by a desktop computer, which can do repetitive calculations very quickly, so that one complete calculation takes only a few seconds. I have not tried to do any of these computations by hand, but I estimate that, with practice, a single iteration of either of the two methods would take an experienced navigator at least an hour using only such old-fashioned tools as tables of logarithms and trigonometric functions.

In his letter to Jefferson, Ellicott had recommended that only moon-sun distances were to be used. Lewis and Clark often ignored this recommendation. They were forced to do so because useful moon-sun measurements are not possible on every day of the month; at other times moon-star measurements are the only alternative. Also, they occasionally made lunar-distance measurements without making any noon or equal-altitudes measurements at all. Such measurements could not be used to determine longitude. They did, however, provide a way to determine the rate of the watch that did not require the observer to remain at a fixed location, although this was not as accurate as the equal-altitudes method if the observer *could* remain at the same longitude. In some cases they probably intended to make the required equal-altitudes measurements the next day, but were prevented by poor weather conditions or by the need to tend to more urgent matters.

Jefferson had specified that Lewis and Clark were to hand over their astronomical measurements to the war office for calculation of the longitudes. In May 1807, after his return to the east coast, Lewis paid Ferdinand Hassler a hundred dollars in advance to make the calculations. Hassler, an astronomer and surveyor, was then the mathematics professor at West Point, and presumably represented the war office. Hassler was the principal organizer and first superintendent of the United States Coastal Survey, so we can assume he was well acquainted with the method of lunars. But he may not have known that he could calculate the missing altitudes or, if he did, he may have mistrusted results obtained by this method. By 1810 he was a professor

at Union College, and was having a terrible time dealing with the Lewis and Clark data. In a letter to Robert Patterson²⁵ he seems to be describing an effort to cope with the missing altitudes by using approximations, when he writes of “calculating backwards from the circumstances & times given [to obtain] what ought to have been observed.” He complains that he has only one of the journals, and that even this is apparently a copy of the original and may well contain errors. After comparing Lewis’s position for the mouth of the Missouri with previously published maps and other information, as well as Lewis’s own data taken farther up the Missouri, Hassler concludes that Lewis’s longitude for the mouth of the Missouri is off by about two degrees. Actually, Lewis’s value agrees better with the modern value than Hassler’s “corrected” version of Lewis’s value. (The custodian of most of the original journals, Nicholas Biddle, realized that Hassler had been given an inadequate map of the Mississippi to work with and, according to a better map in his possession, Lewis had the position about right. This opinion was relayed to Hassler.)²⁶

By 1817 Hassler had abandoned the calculations “in despair.”²⁷ The reason for Hassler’s despair is not clear. As previously noted, Patterson’s Problem 4th described a procedure that was widely known and was, in fact, a valid procedure yielding reliable results. Why did no one convince Hassler that many of the Lewis and Clark data were completely adequate? Robert Patterson and Andrew Ellicott, who were Lewis’s instructors in celestial navigation and fellow members with Hassler of the American Philosophical Society, were in a position to do so.

Throughout his career Hassler made many influential friends and many influential enemies.²⁸ Patterson was one of the friends. Not only did he recommend Hassler for the Coastal Survey position, but even as late as 1815 he wrote warm letters of recommendation for him to the former president, Thomas Jefferson, and to President James Madison.²⁹ Patterson knew from Hassler’s 1810 letter about his trouble with the calculations. Perhaps he gave Hassler advice that Hassler was unable or unwilling to use, or perhaps Patterson was convinced by Hassler’s letter that the Lewis and Clark data were useless and thought it best to let the matter drop.

As to whether Ellicott could or would have helped Hassler, one can only speculate. On several occasions in later years Hassler published disparaging remarks about Ellicott’s professional ability and accused Ellicott of trying to damage his reputation,^{29,30} so it may be there was already some animosity between them at this time. Ellicott, as an army major and Lewis’s mentor, might have wanted to do the longitude calculations for the war office, which is where Jefferson had directed the calculations be performed. But it was Hassler, as a professor at West

Point and a civilian, but also an employee of the war office, who got the job. Thus, Ellicott might not have been inclined to help Hassler. In fact, Ellicott may even have resented Hassler's appointment to the West Point professorship: at about this time Hassler was forced to resign his West Point position because Congress had supplied no funding for it;³¹ when funding eventually was provided, it was Ellicott who was appointed to Hassler's old position. But any imputation of malice to Ellicott is pure conjecture, to which the rejoinder must be that Ellicott would have had an interest in seeing a useful outcome of his work with Lewis, and this would have motivated him to be helpful if he had been aware of Hassler's difficulties.

JEFFERSON'S DISAPPOINTMENT

The idea for the Lewis and Clark expedition originated with President Thomas Jefferson, who was personally involved in all phases of the planning for it. Jefferson was interested in everything, and he instructed the explorers to return information on a wide variety of subjects: botany, zoology, geography, geology, meteorology, Indian customs and languages, and anything else. Lewis and Clark obliged him by keeping voluminous and detailed journals that take up a foot of shelf space in a printed edition. As late as 1817, with Hassler out of the picture, Jefferson, still hopeful, was waiting for a secretary of war to be appointed so that he could "propose to him to have made, at the public expence, the requisite calculations, to have the map corrected in its latitudes and longitudes, engraved and published on a proper scale."³² As we have seen, the corrections never became available and Jefferson remained forever disappointed. The considerable effort that Lewis and Clark put into making usable, high-quality astronomical measurements was totally wasted.

COMPUTER CALCULATIONS OF POSITION FROM THE LEWIS AND CLARK DATA

For each set of lunar-distance readings the table shows

- the date of the observation and the celestial partner for the lunar-distance measurement
- the modern values of latitude and longitude for the point of observation
- the latitude calculated from the measured altitude of the sun at high noon and the average of the corresponding longitudes calculated using Bergantino's version of the Problem 4th method

- the average values of latitude and longitude calculated by Ellicott's 1803 method, using only the lunar-distance and equal-altitudes measurements
- the number of lunar-distance readings in the set
- the error of the average longitude calculated for the set
- the probable error of the average longitude calculated for the set

Calculation of a longitude requires equal-altitudes measurements and lunar-distance measurements, all made at the same location. Some of Lewis's data meet these requirements, but, as it happens, none of Clark's do. The table gives results calculated from observations made from June 1804 to August 1805. Lewis usually made a succession of measurements of the lunar distance to a particular celestial body, spaced a few minutes apart. Often he made as many as twenty measurements, and on one occasion forty-eight measurements in one set.

My computer programs for both the 1799 and the 1803 methods make a complete calculation for each lunar-distance measurement within one set, and then take an unweighted average. Only this average for each set is presented in the table. (At the time of Lewis and Clark, only one calculation would have been made for each set using the average of the lunar distances and the average of the times for the set.) Although the results calculated from moon-star distances are usually wide of the mark, the fourteen moon-sun results are much better, with an rms deviation from the modern values of 35 minutes of longitude (about 30 miles at latitude 40°). This is about standard for the method, and refutes the accepted opinion that the Lewis and Clark astronomical observations were of poor quality.

The probable error of the mean longitude is an estimate of the consistency of the longitudes calculated from that set of measurements. This is not a measure of the validity of the longitude calculated for the set because it makes no allowance for any systematic errors due to such factors as improper calibration of the sextant, incorrect watch rate, and observer bias in measuring with the sextant. Except for one set of only two measurements (10 July 1805, to α Pegasi), the average of the probable errors of the longitude is only 5'. This indicates that if the systematic errors in Lewis's measurements could have been eliminated, or could be allowed for in the calculations, the calculated longitudes would be much nearer to the correct values.

As mentioned earlier, Lewis made several measurements at Fort Mandan to determine the longitude of the Mandan villages. Usable lunar distances were measured on 6 February and 23 February 1805 in the usual fashion, but earlier a lunar eclipse provided a different possibility for determining Greenwich time and the longitude. Arlen J.

<i>Observation</i>	<i>Modern values^a</i>	<i>Latitude & longitude</i>		<i>Number of lunar distance readings</i>	<i>Error in longitude</i>	<i>Probable error of the mean</i>
		<i>Lewis & Clark/ Bergantino^b</i>	<i>Ellicott^c</i>			
2 June 1804 Moon to sun	38° 36' 0"N 91° 57' 0"W	38° 31' N 91° 47' W	38° 35' N 91° 47' W	26	-10'	4'
3 June 1804 Moon to sun	38° 36' 0"N 91° 57' 0"W	38° 31' N 91° 30' W	39° 1' N 91° 36' W	36	-27'	5'
29 June 1804 Moon to sun	39° 7' 12"N 94° 36' 0"W	39° 5' N 94° 8' W	39° 7' N 94° 8' W	48	-28'	2'
30 June 1804 ^d Moon to sun	39° 10' 48"N 94° 40' 12"W	39° 7' N 94° 18' W	39° 7' N 94° 18' W	20	-22'	5'
11 July 1804 Moon to Spica	40° 1' 48"N 95° 22' 48"W	39° 56' N 93° 40' W	39° 24' N 93° 52' W	14	-102'	5'
12 July 1804 Moon to sun	40° 1' 48"N 95° 22' 48"W	39° 56' N 94° 51' W	39° 26' N 94° 57' W	16	-32'	4'
17 July 1804 Moon to Spica	40° 32' 24"N 95° 39' 0"W	40° 27' N 93° 11' W	41° 1' N 93° 6' W	8	-148'	7'
22 July 1804 Moon to Antares	41° 10' 48"N 95° 51' 36"W	41° 3' N 94° 30' W	41° 10' N 94° 31' W	6	-81'	5'
27 July 1804 Moon to α Aquilae (Altair)	41° 10' 48"N 95° 51' 36"W	41° 6' N 96° 24' W	41° 46' N 96° 24' W	6	+33'	3'
13 Aug. 1804 Moon to sun	42° 19' 48"N 96° 21' 36"W	No noon observation	42° 18' N 95° 19' W	26	-62'	4'
31 Aug. 1804 Moon to sun	42° 51' 0"N 97° 27' 36"W	No noon observation	42° 51' N 98° 43' W	6	+76'	11'
6 Feb. 1805 Moon to sun	47° 16' 54"N 101° 16' 24"W	47° 21' N 100° 37' W	47° 14' N 100° 34' W	12	-39'	3'
23 Feb. 1805 Moon to sun	47° 16' 54"N 101° 16' 24"W	47° 21' N 101° 46' W	47° 18' N 101° 46' W	12	+30'	1'
3 April 1805 ^e Moon to sun	47° 16' 54"N 101° 16' 24"W	47° 21' N 101° 0' W	47° 31' N 100° 59' W	7	-16'	6'
22 April 1805 Moon to sun	48° 6' 22"N 103° 39' 0"W	No noon observation	48° 7' N 103° 36' W	16	-3'	3'
9 June 1805 Moon to α Aquilae (Altair)	47° 55' 45"N 110° 29' 4"W	47° 29' N 108° 31' W	47° 37' N 108° 32' W	6	-118'	10'
9 June 1805 Moon to Spica	47° 55' 45"N 110° 29' 4"W	47° 29' N 108° 25' W	47° 37' N 108° 24' W	6	-125'	6'
10 July 1805 ^f Moon to Antares	47° 27' 23"N 111° 17' 36"W	47° 3' N 108° 12' W	47° 17' N 108° 14' W	8	-185'	2'

(Continued)

Observation	Modern values ^a	Latitude & longitude		Number of lunar distance readings	Error in longitude	Probable error of the mean
		Lewis & Clark/ Bergantino ^b	Ellicott ^c			
10 July 1805 ^{d,g} Moon to α Pegasi (Markab)	47° 27' 23"N	47° 3'N	47° 17'N			
21 July 1805 Moon to sun	46° 42' 15"N 111° 48' 10"W	No noon observation	46° 52'N 112° 3'W	2	-171'	41'
29 July 1805 Moon to sun	45° 55' 27"N 111° 31' 10"W	45° 24'N 111° 19'W	45° 56'N 111° 12'W	20	-12'	1'
29 July 1805 Moon to Antares	45° 55' 27"N 111° 31' 10"W	45° 24'N 109° 59'W	45° 59'N 109° 45'W	10	-92'	11'
20 Aug. 1805 ^h Moon to Sun	44° 59' 25"N 112° 51' 52"W	44° 40'N 112° 56'W	44° 52'N 112° 56'W	15	+4'	2'

a. From a list compiled by Robert Bergantino (private communication).

b. Latitude from Lewis & Clark noon observation; longitude by Bergantino's iterative method.

c. Latitude and longitude by Ellicott's method of 1803.

d. According to the journal entries, the location appears to be unchanged from 29 June, so the latitude and equal-altitudes measurements from that day were used in the calculations for this date.

e. The recorded moon-to-sun distances for 3 April 1805 all lie between 43° and 44°. According to the *Nautical Almanac* for that year no such values were possible at any time on that day. When I tried changing 43° and 44° to 53° and 54° in the recorded lunar distances, the result was so close to the modern value that I included it in the table, despite the doubtful propriety of such an emendation of the data.

f. The equal-altitudes data for 10 July 1805, as listed in Lewis's journal, are in bad shape. The altitude and some of the time readings are identical with the altitude and the corresponding times listed for 3 July.

g. On 10 July 1805 Lewis made lunar-distance measurements on two stars, either of which might have been α Pegasi. He wrote that the star in his second set of measurements was the more probable candidate. In agreement with his opinion, the longitude I calculate from the second set of measurements is reasonable, but that from the first set is meaningless. Only the results for the second star are given in the table.

h. For the calculations of the longitude for the moon-Antares observations of 20 August 1805, I followed Bergantino in discarding all of the recorded distance readings except for readings 4, 5, and 6. For the latitude calculation I used the value he derived from the Lewis and Clark noon altitude of the sun.

Large³³ has given a comprehensive discussion of the significance of the lunar eclipse observations. Briefly stated, David Thompson had determined the longitude of this place some years earlier, and had reported his result as 101° 25'W. Lewis's lunar eclipse result was 99° 26' 45"W, and the correct longitude is 101° 16' 24"W. Thus, in hindsight, we see that Thompson's lunar-distance measurement was better than Lewis's lunar eclipse measurement. But Lewis's two lunar-distance measurements yield longitudes of 100° 37'W and 101° 46'W. These are much closer to the mark, and their average is very close to the correct value.

There do appear to be significant systematic errors in Lewis's data. Except for the first three readings in 1805, the latitudes he calculated from the noon altitudes of the sun (third column) are consistently low by about 5'. Either he was using the wrong value of the index error of the octant (which was always used for these measurements) or there

was a systematic personal bias in his measurements. Fortunately, in most cases, the resulting value of the longitude is rather insensitive to the choice of latitude used for the calculation, as can be seen from the fairly consistent agreement between the longitudes calculated with and without use of latitudes obtained from noon measurements (third and fourth columns). Similarly, there is a pronounced tendency for the calculated longitudes to be too far east. It is especially conspicuous that all but one of the moon-star longitudes are too far east by more than a degree. Of course the reason for this cannot be determined with certainty, but two possible explanations are use of an incorrect value of the index error of the sextant, and a personal bias in making the measurements.

The observer does not measure distances to the center of the moon, but to the nearest or farthest point (the nearer or farther "limb") of the moon. The distance of a star to the center of the moon is then found by adding or subtracting the known radius of the moon, as appropriate. While looking through the sextant, the observer sees a direct image of the star and, superimposed on it, a view of the moon that originates as a reflection from a tiltable mirror. As the tilt of the mirror is adjusted, the reading on the sextant scale changes. When the tilt of the mirror is adjusted to bring the image of the star just into contact with the limb of the moon's image, the reading on the scale (after correction for index error) is the angular distance from the star to the limb of the moon. To this the moon's radius must be added or subtracted, and the result corrected for parallax and refraction.

First, consider the effects of improperly correcting for index error. If a star is west of the moon and the sextant reading after correction for index error is too small because of a wrong value of the index error, the Greenwich time read from the *Nautical Almanac* will be too early and the calculated longitude will be too far east. Conversely, if the star is east of the moon and the corrected sextant reading is again too small, the calculated longitude will be too far west. For the longitudes I have calculated from moon-star distances, the star was west of the moon in six cases for which the average error in the longitude is about 110' east, and east of the moon in three cases for which the average error in longitude is again about 110' east. Thus, improper correction for index error is not the main problem.

Next, consider a personal bias such that the observer always thinks that, as seen in the sextant, a star is exactly touching the limb of the moon when, in fact, there is a small gap between the two. If the measurement is to the west side of the moon, which will be the case in the early evening, the calculated Greenwich time will be too early and the longitude too far east. If the measurement is to the east side of the

moon, which will be the case in the early morning, the calculated Greenwich time will be too late and the calculated longitude too far west. These results do not depend on whether the star itself is east or west of the moon. And, indeed, the eight evening moon-star longitudes are all too far east, by an average of about $110'$, while the one morning moon-star distance (27 July 1804) yields a longitude that is too far west by $33'$. Examination of the results from the moon-sun measurements reveals similar tendencies, but of much smaller magnitude. This indicates that just such a personal bias has influenced Lewis's data.

For Lewis's observations it would take an error of about 3 minutes of arc in the observed lunar distance to produce a shift as big as $110'$ in the calculated longitude. Considering that the diameter of the moon is about $30'$, this is an error equivalent to 10 percent of the moon's diameter. It may be difficult to understand how a gap this large between star image and moon image could be mistaken for no gap at all. Part of the problem may be that, in lunar distance measurements, it is impossible to watch the moon and star images moving slowly and steadily toward or away from each other along the line separating their centers. The observer must superimpose an additional small, vibratory twisting motion to his sextant to make the two images swing back and forth in opposite directions along a line perpendicular to the line of centers. The object of this is to note the instant at which the two images just brush each other during one of these oscillations. That the sextant often has to be held in a very awkward position compounds the difficulty of this process.

The rather poor results from these moon-star measurements justify Ellicott's 1803 recommendation that only moon-sun distances be measured. There are other arguments supporting this recommendation: not only is it easier to judge when the moon and sun images are just brushing past each other, but moon-sun distance measurements are made in the daytime when it is easier to read the watch and the sextant scale and to record the readings than at night by the light of a lantern.

The low values of latitude obtained from noon altitude measurements, mentioned previously, cannot be explained by an observer bias of the type just described. If the observer judges that the lower limb of the sun is just touching the upper limb of the reflection of the sun in the artificial horizon when, in fact, there is actually a gap, the altitude of the sun obtained from this measurement will be too low, and the resulting calculated latitude too high. Since the actual case is just the opposite, the best explanation of the low latitude values appears to be an incorrect value of the index error of the octant that Lewis and Clark always used for their noon measurements for latitude.

ACKNOWLEDGMENTS

In order to make my calculations I had to have information from the 1804 and 1805 editions of the *Nautical Almanac*. This was difficult to get because existing copies for those years are scarce and are treated as rare books. Fortunately I have been provided with photocopies of relevant pages from several editions of the *Nautical Almanac* for those years by Robert Bergantino of Butte, Montana, and by Brenda Corbin of the U.S. Naval Observatory Library, Washington, D.C., who also provided copies of important pages from an early edition of Bowditch's *New American Practical Navigator*, Maske-lyne's *Tables requisite*, and Kelly's *Practical Introduction to Spherics and Nautical Astronomy*, as well as from other sources important for understanding early nineteenth-century navigational astronomy. With the understanding I have gained and with the *Nautical Almanac* tables available I have been able to write computer programs and enter the requisite astronomical data for particular dates into computer files for use by the programs. Guy Benson, of Raleigh, N.C., provided me with useful documents including a copy of Robert Patterson's astronomical notebook. With permission from the Smithsonian Institute, the Library of the American Philosophical Society provided a photographic copy of Andrew Ellicott's astronomical journal. I have also benefited from other information supplied by Robert Bergantino, Bruce Stark of Eugene, Oregon, and Gary Moulton, editor of *The Journals of the Lewis and Clark Expedition*.

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23. To see that this is so one may picture an observer at a certain latitude who measures the altitude of a star exactly seven hours after his local noon. A second observer 15° (one hour) west of the first observer but at the same latitude will see the same star at almost the same altitude one hour after the first observer does. This will be the same length of time, seven hours, after his own local noon. A third observer at the same latitude but 45° (three hours) west of the first will see the same star at nearly the same altitude three hours after the first observer does, which will again be seven hours after his own local noon. The conclusion from this is that at a given latitude the observed altitude of a given star depends more on the time from noon than on the observer's longitude. Thus, in calculating the approximate values of the missing altitudes, the important quantity is the elapsed time since local noon, which the navigator has from his watch readings. Approximate altitudes calculated this way are best for the stars and planets, and worst for the moon.
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